Survey Analysis
week 4
“Stratified and cluster sampling”

What have we done so far

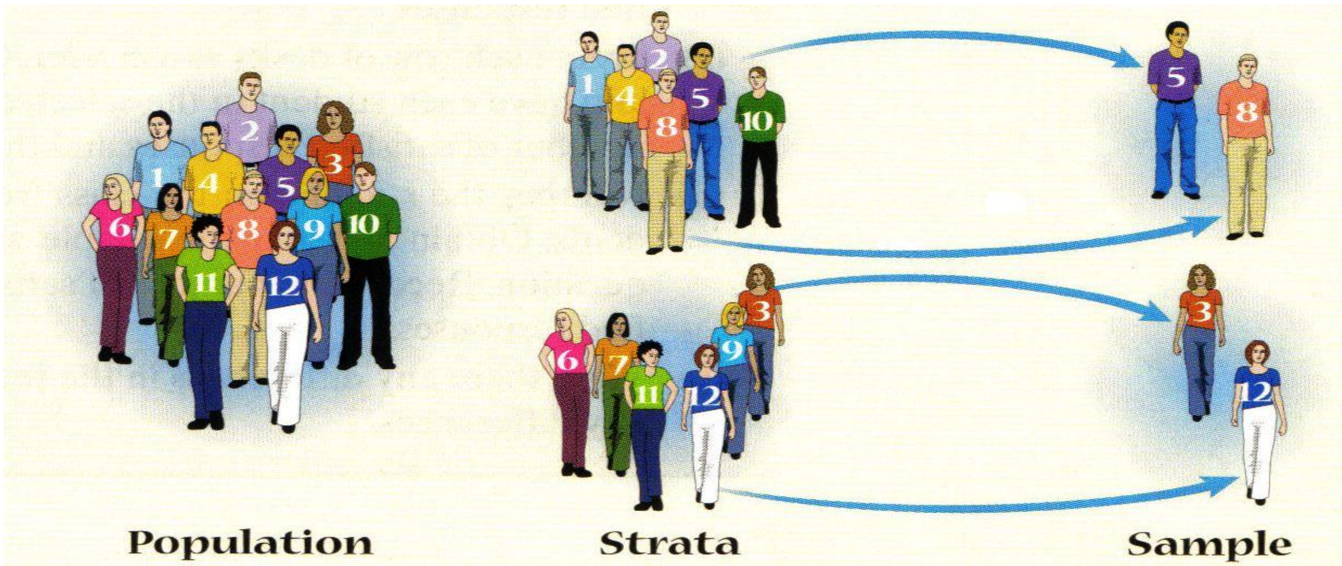
- Basics of survey error (TSE, bias, error)
- Mean, variance, standard error, proportions
  - SRS sampling – with and without replacement
  - n-1
  - Fpc
- R – survey and sampling packages
- Sample size calculations for mean
  - For simple random sample
  - Given CV, margin of error, Confidence interval
  - Given alpha, Beta, population variance

What will we do today

- Discuss THE week 3 in groups
  - Questions, issues?
- Lecture on stratification and clustering
  - Stratification and clustering: why?
  - Variance estimation, design effects
- 2 short class exercises
  - Set up svydesign objects

Stratified Random Sampling

- Population exists of pre-defined groups
- Use this information to optimize sampling design
- Idea – take a SRS from each group (*stratum*) and combine these for the final sample



Stratified sample design – why?

A fictitious population of companies in Estonia

Sector/size (employees)	1-4	5-49	50+
Manufacturing	7000	1.800	335
Trade	14.000	3.000	250
Business	20.000	3.335	335
Government/education	1.600	282	79
Healthcare	1.100	625	224
Agriculture	682	118	3

Stratified sample design – why?

1. Increase precision in total (< s.e.)
2. Ensure precision in subgroups
3. More practical when sampling frames are only available per subgroup
4. + to limit other surveys errors (later)

Sector/size (employees)	1-4	5-49	50+
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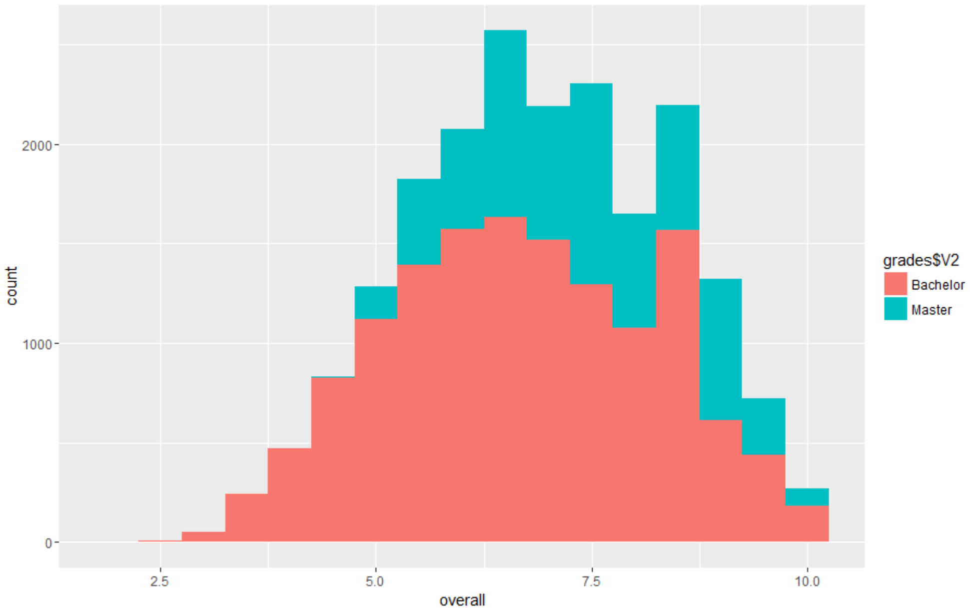
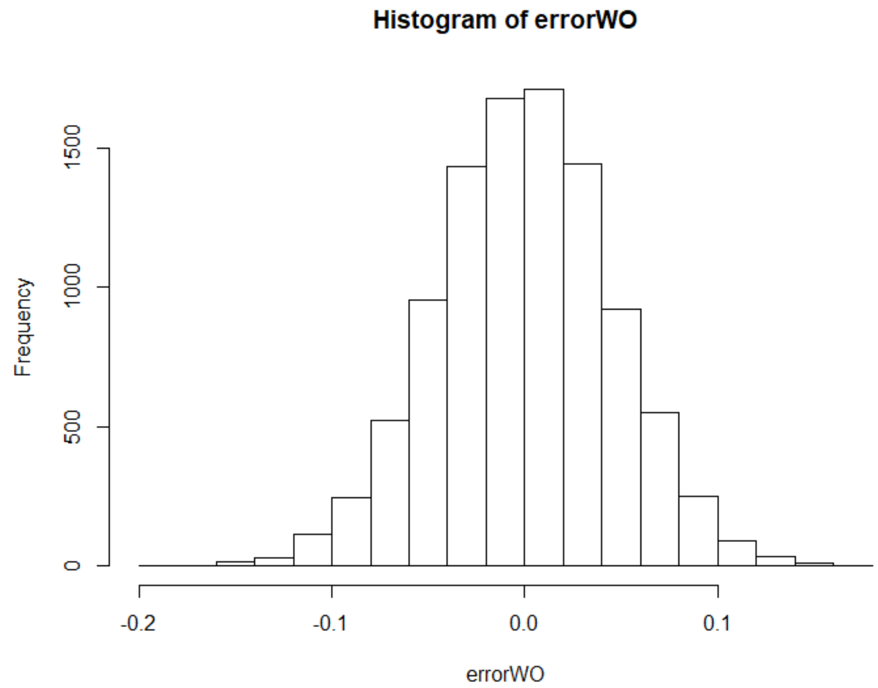
An example: population of student grades at UU



- Total: 20000 students
  - 14000 BA, 6000 MA
- Now, what if we want to sample?
  - Last week: simple random sample
  - Size = 1000
- R-Code of example on BlackBoard

Simple Random sample

- Draw 10000 samples (replications)
  - Each, n=1000



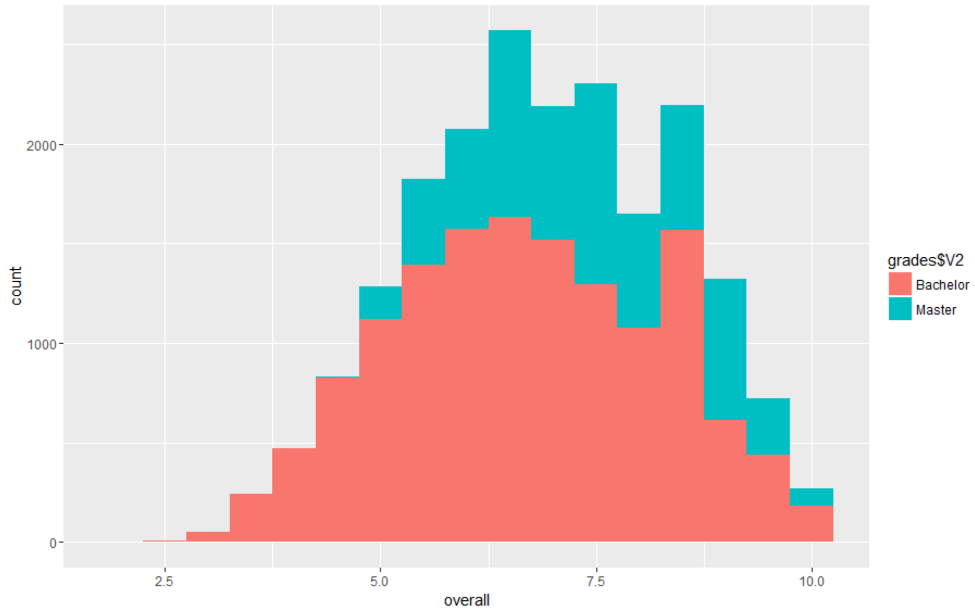
Simple Random sample

- Size=1000 out of 20000
  - Draw 10000 samples (replications)
  - Roughly 700 BA students, 300 Ma students

	Bias	Variance (s.e)
Simple random sample	-.026	.0021

- MSE = $-.026^2 + .0021 = .0027$
- How can we improve?

Back to example: population of student grades at UU

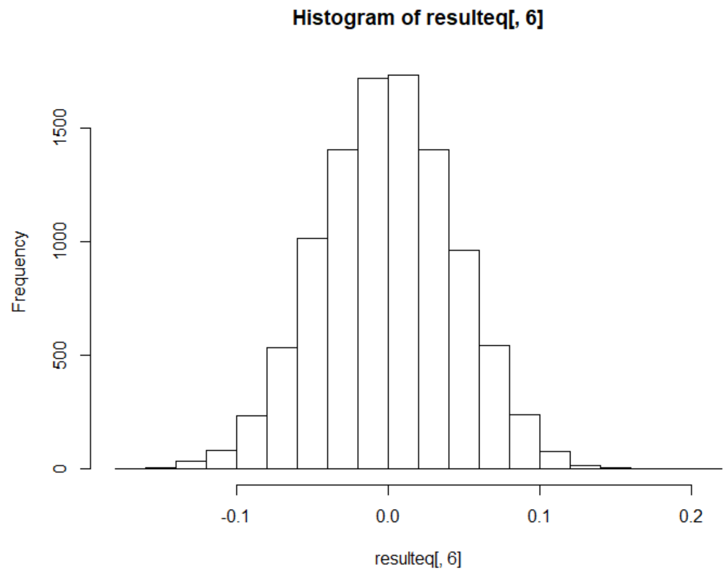


- Total: 20000 students
  - 14000 BA, 6000 MA
- Why stratify on degree?
  - Mean BA: 6.69
  - Mean MA: 7.41
  - Var(BA):2.36
  - Var(MA): 1.49
  - Var(total): 2.20

R-Code of example on BlackBoard

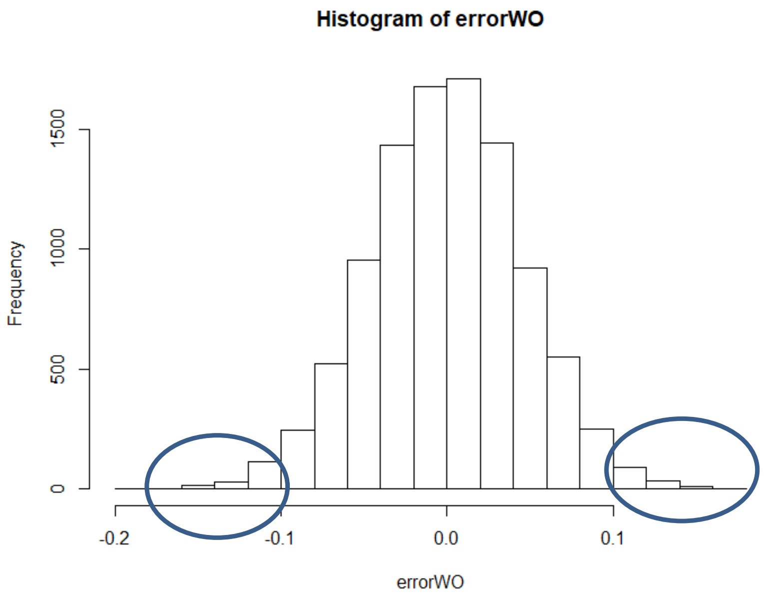
Stratification with equal probabilities

- Size=1000, 10000 replications
  - Exactly 700 BA students, 300 Ma students
  - Proportional to Size (PPS)
  - Error compared to population mean

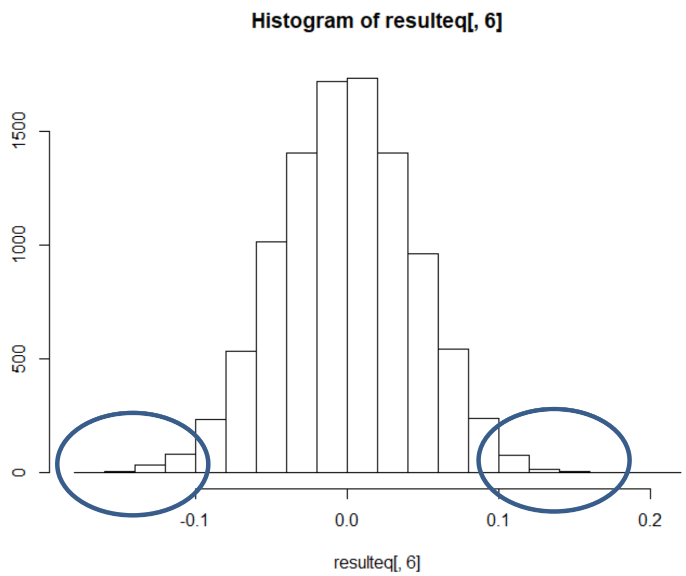


Comparison of standard errors

SRS



Stratification pps



Slightly fewer extreme samples: stratification reduces s.e.

Stratification with equal probabilities

- Size=10000
  - Exactly 700 BA students, 300 Ma students
  - Proportional to Size (PPS)

	Bias	Var(mean)	Var(mean) Bachelor	Var(mean) Master
Simple random sample	-,026	,0021		
Stratification equal probabilities	-,031	,001966	,0031	,0047

Formulas

- 1. Mean with SRS in every stratum
- $\bar{y}_h = \frac{1}{n_h} \sum y_{hj}$

- 2. Combining means :

$$\bar{y}_{\text{str}} = \sum \frac{N_h}{N} \bar{y}_h$$

Size of stratum in population

Formulas

- 1. Mean with SRS in every stratum (pps)

$$\bar{y}_h = \frac{1}{n_h} \sum y_{hj}$$

- 2. Combining means (pps):

$$\bar{y}_{\text{str}} = \sum \frac{N_h}{N} \bar{y}_h$$

- 3. Variance (pps):

- $\hat{V}(\bar{y}_{\text{str}}) = \sum (1 - \frac{n_h}{N_h}) \bar{y}_h \, (\frac{N_h}{N})^2 \, \frac{s_h^2}{n_h}$

Variance in stratum

Fpc per stratum

(Size of stratum in population)^2

Design effect

- If we have a measure of the bias in standard (SRS) estimates of the sampling variance this can be used to adjust the variances or standard errors from SRS
- $D_{eff} = \dfrac{\text{Variance under specific design}}{\text{Variance under SRS}}$
- D_{eff}= inflation factor for variance
  - Can be used for estimating effective sample size
  - $n_{eff} = N/D_{eff}$
- $D_{eft} = \sqrt{D_{eff}}$ = inflation for standard error

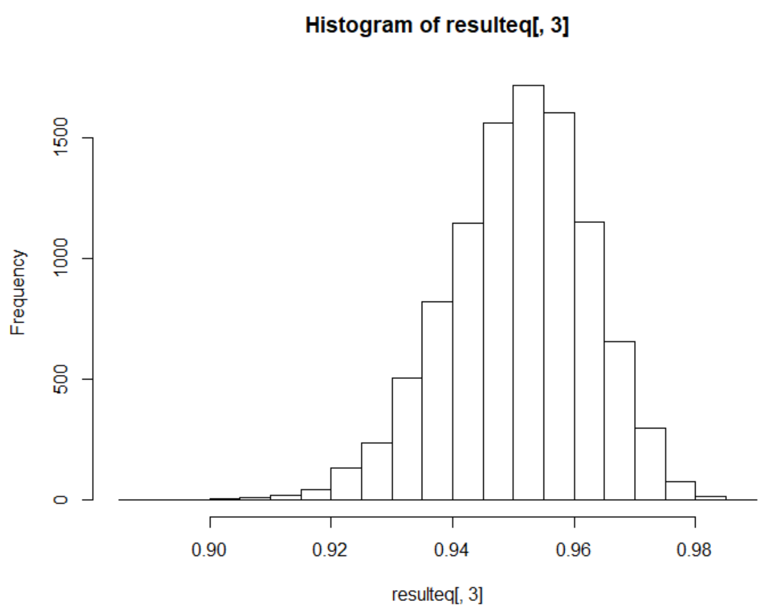
Design effect (2)

- D_{eft} = .00197/.0021 = .9523
  - D_{eff} = $.952^{2.}$ We need .906 as many respondents for same precision
  - N_{eff} = 1000/.906 = 1103
- Stratified sample: D_{eff} <1
- Cluster sample: D_{eff} >1

- R produces Design effect as well
  - For one sample against same sample under SRS
  - (Pooled) d_{eft} in R: .951
    - Rounding error in manual computation above

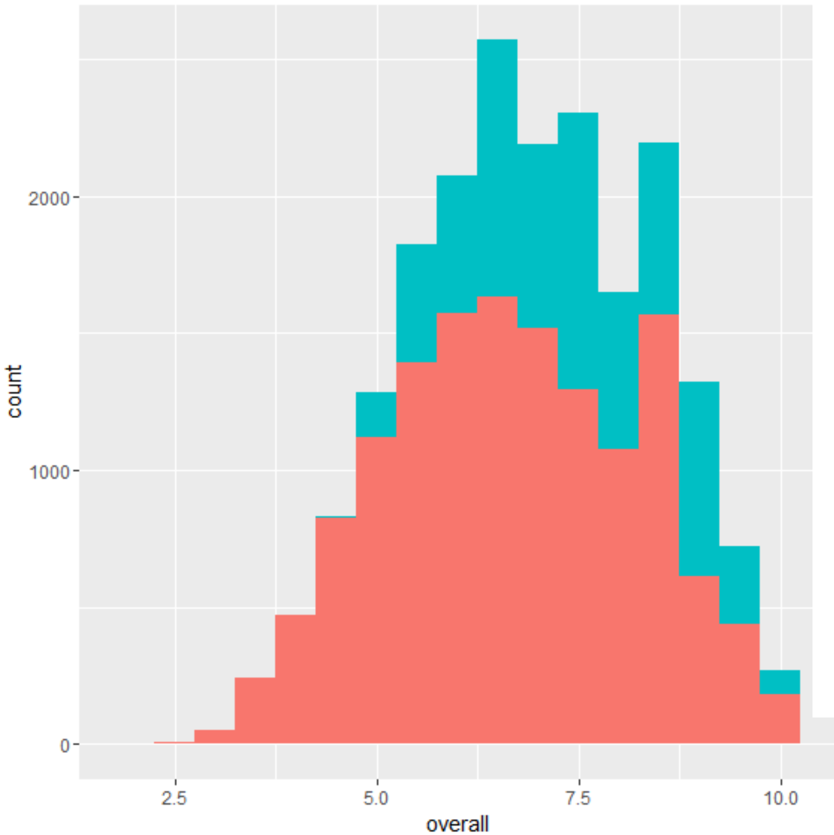
Design effect (3)

- R produces Design effect as well
  - 10000 estimates of design effect



  - Note this is skewed (why?)

Unequal probabilities



- Why?
  - Increase precision for small group in population (Ma students
  - Optimize the precision for the total mean grade?

Unequal probabilities

- Population: 14000 BA, 6000 MA
  - We can oversample MA students
    - Because there is more nonresponse, or we want increased precision in this group
  - Imagine we sample 500 students from each group
    - $\pi(BA) = 500/14000 = .035714$
    - $\pi(MA) = 500/6000 = .083333$

Stratification with unequal probabilities

- Size=1000
  - 500 BA students, 500 Ma students

	Bias	Var(mean)	Var(mean) Bachelor	Var(mean) Master
Simple random sample	-,026	,0021		
Stratification PPS	-,031	,001966	,0031	,0047
500 in each	,023	,0025	,0047	,0027

- #$%&: wasn't stratified sampling supposed to decrease the variance?

Optimal allocation

- How large should samples within strata be so that s.e. for population is minimized?
- Neyman allocation:
  - N_h = sample size stratum
  - S_h = standard deviation in stratum

$$n_h, Neyman = \frac{N_h S_h}{\sum N_l S_l}$$

- Needed?
  - N_l = Population size
  - S_l = Standard deviation in population (!)
- Often costs are included (more complicated)

Optimal allocation: example

- Neyman allocation:
  - N_h = sample size stratum
  - S_h = standard deviation in stratum

$$n_h, Neyman = \frac{N_h S_h}{\sum N_l S_l}$$

- Population variance: 2,20
- Stratum variances : 2,36 and 1,49
  - See slide 7

- Stratum standard deviations = $\sqrt{2,36}\ and\ \sqrt{1,49}$

Optimal allocation: example

- Neyman allocation:
  - N_h = sample size stratum
  - S_h = standard deviation in stratum

$$n_h, Neyman = \frac{N_h S_h}{\sum N_l S_l} n$$

$$N_{MA} = \frac{(6000 * \sqrt{1.49})}{(6000 * \sqrt{1.49}) + (14000 * \sqrt{2.20})}$$

$$= \frac{7324}{28831} = 0.254 * 1000$$

- $N_{BA} = \frac{(6000 * \sqrt{1.49})}{(6000 * \sqrt{1.49}) + (14000 * \sqrt{2.20})}$

$$= \frac{21507}{28831} = 0.746 * 1000$$

Optimal allocation to strata

- Size=1000
  - 746 BA students, 254 Ma students

	Bias	Var(mean)	Var(mean) Bachelor	Var(mean) Master
Simple random sample	-,026	,0021		
Stratification equal probabilities	-,031	,001966	,0031	,0047
500 in each	,023	,0025	,0047	,0027
Neyman	,04	,001962	,0029	,0055

Optimal allocation to strata

- Unequal selection probabilities
  - 746 BA students, 254 Ma students
  - D_{eft} = .01962/ .0021 = .934
  - n_{eff}= 1000/$.93^2$= 1145
- In words: we optimize stratification when:
  - The stratum accounts for a large part of the population
    - BA = 0.7, MA= 0.3
  - The variance within the stratum is large; we sample more heavily to compensate
    - variance BA:2.36, Variance MA: 1.49
  - -> leads to 746 BA students

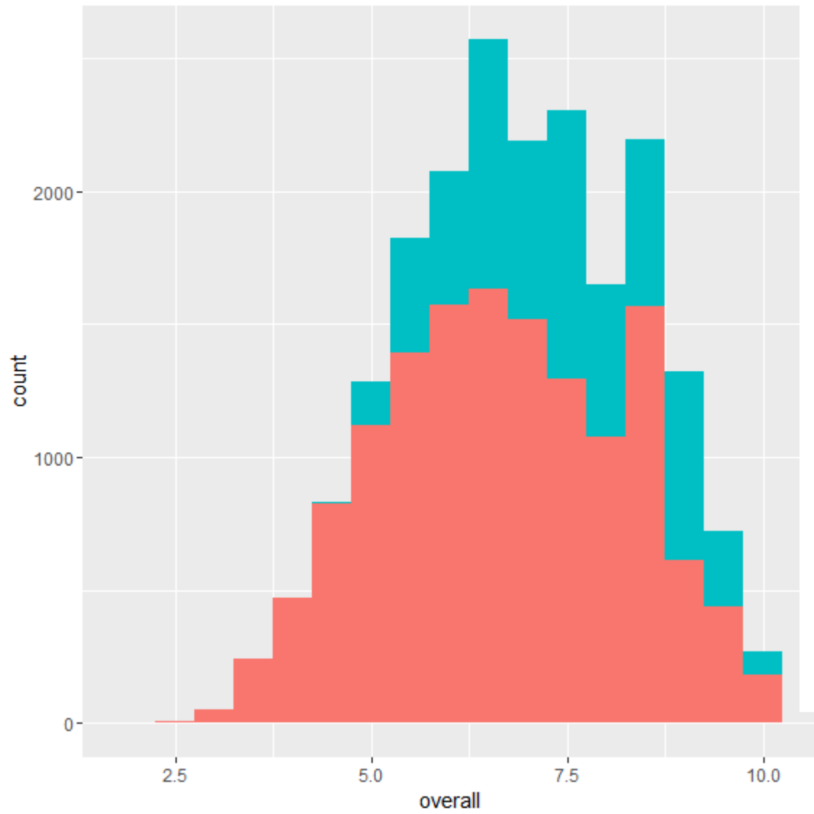
2 ways to deal with unequal selection

- 1. We can combine means from n_h strata
  - If we know the population sizes (see earlier slides)
- 2. We can use sampling weights
  - With SRS, sampling weights are equal for all i
  - In Neyman allocation example:
    - $\boldsymbol{\pi}_{ba} = \frac{746}{14000} = .053$. Sampling weight $w_{ba} = \frac{1}{.053} = 18.76$
    - $\boldsymbol{\pi}_{MA} = \frac{254}{6000} = .042$. Sampling weight $w_{ma} = \frac{1}{.042} = 23.61$
  - $\bar{y}_w = \frac{\sum w_i y_i}{\sum w_i}$

Bringing Nonresponse in
(more in week 8-10)

- We often oversample from specific groups
  - Nonresponse
  - Imagine:
    - Response rate of 50% among BA students
    - And 20% among Ma students
  - Achieved sample = 399 BA , 40 MA students
    - We can correct by using Nonresponse weights, but this is inefficient

Should we actually stratify on Ba/MA?



- What variable to stratify on?
  - BA/MA, Faculties, Programme
    - Social science, humanities, etc.
  - Gender, Age, living in Utrecht
  - Member of student union
  - Etc.
- We can stratify on multiple variables
- Survey mode:
  - E-mail: stratification easy
  - Face-to-face: costs!
    - Cluster sampling?

Class exercise 1

- Draw and analyse a stratified sample from a simplified dataset
  - Understand the basic R code for specifying **stratified** survey design object
  - Which variable to stratify on?
- See Class exercise document (Blackboard)

Adding in clusters

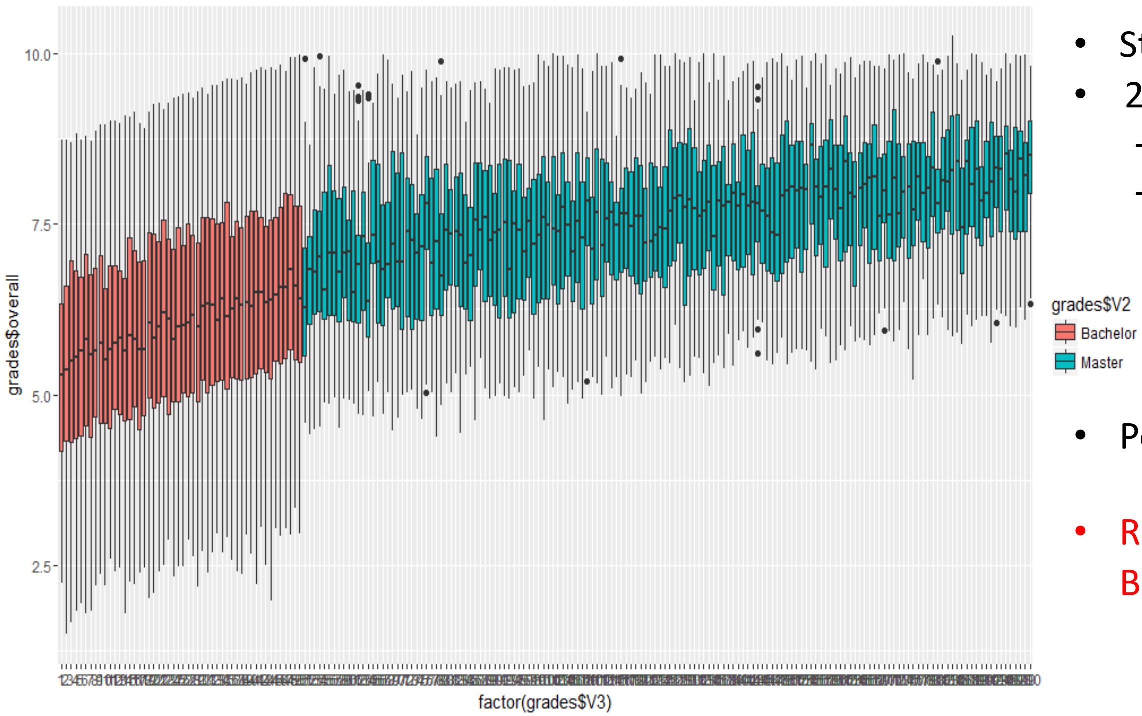
- What if we don't use e-mail?
  - Cost per case: € 0,50
- But face-to-face
  - Costs per programme: € 50,-
  - Costs per case: €10,-

- Households, businesses
- Hospitals, schools
- Towns!

Terminology

- Primary sampling units (psu)
- Clusters
- Secondary sampling units (ssu)
- Intra-Cluster(Class) Correlation Coefficient (ICC) One-stage sample: whole cluster interviewed
- Two-stage sample: further sampling within clusters
- Ratio estimation (week 7)

Example – 150 programmes (Ba/MA)
simulated data



- Student grades (y)
- 200 programmes (x)
  - 50 BA, n=280 each
  - 150 MA, n=40 each

- Population mean: 6.52

- R-code is available on Blackboard

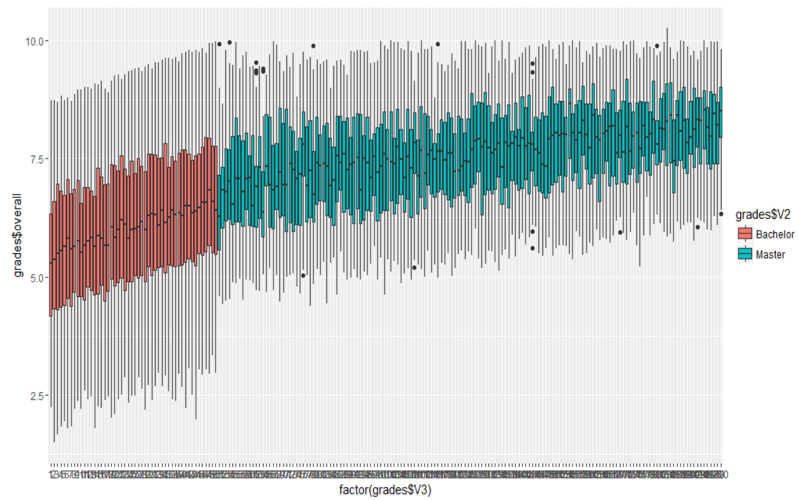
Cluster sampling – why and how

- How?
  - One-stage: ask everyone in cluster
    - Educational surveys: ask every pupil at school (or class)
  - Two stage: do a sample within every cluster
  - Multi-stage: e.g.
    - 1. stratify on income in neighboorhood
    - 2. neighbourhoods (PSU)
    - 3. households (SSU)
    - 4. individual in hh

Why **not** do a cluster sample

- It is inefficient from a statistical perspective
  - ICC: measure of relative variance between clusters

$$ICC = \frac{s_b^2}{(s_b^2 + s_w^2)}$$



- S^2_b: Variance between clusters
- S^2_w: Variance within clusters

One-stage cluster sampling

Clusters of equal size:

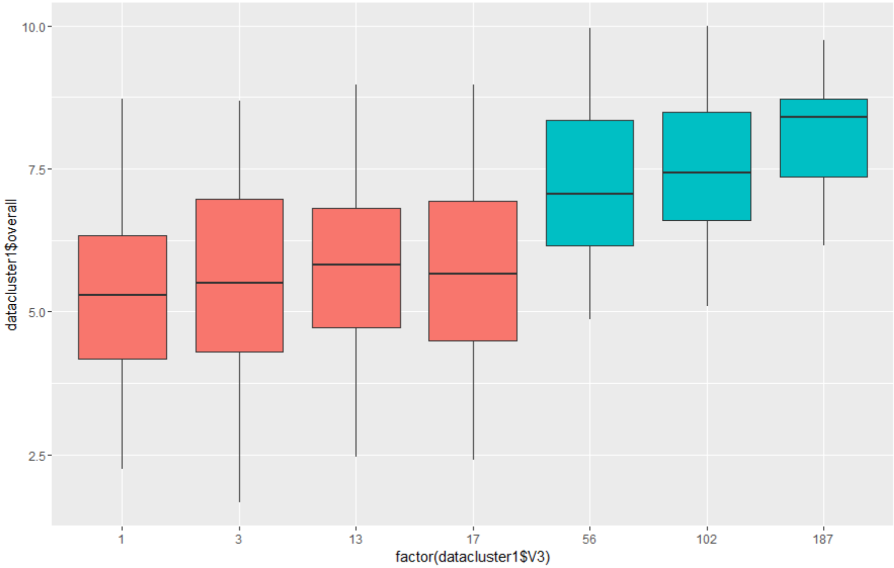
- Selection probabilities equal for all i -> SRS of clusters
- Rarely the case

Clusters of unequal size:

1. draw clusters with unequal selection probabilities
   – Proportional to Size (PPS)
   – Self-adjusting sample
2. Or, draw SRS of clusters
   – Use weights to correct for unequal probabilities

- Example: draw 7 clusters out of 150 with SRS
  – Average cluster size = 133, expected sample size = 933

1 draw for One-stage cluster sample



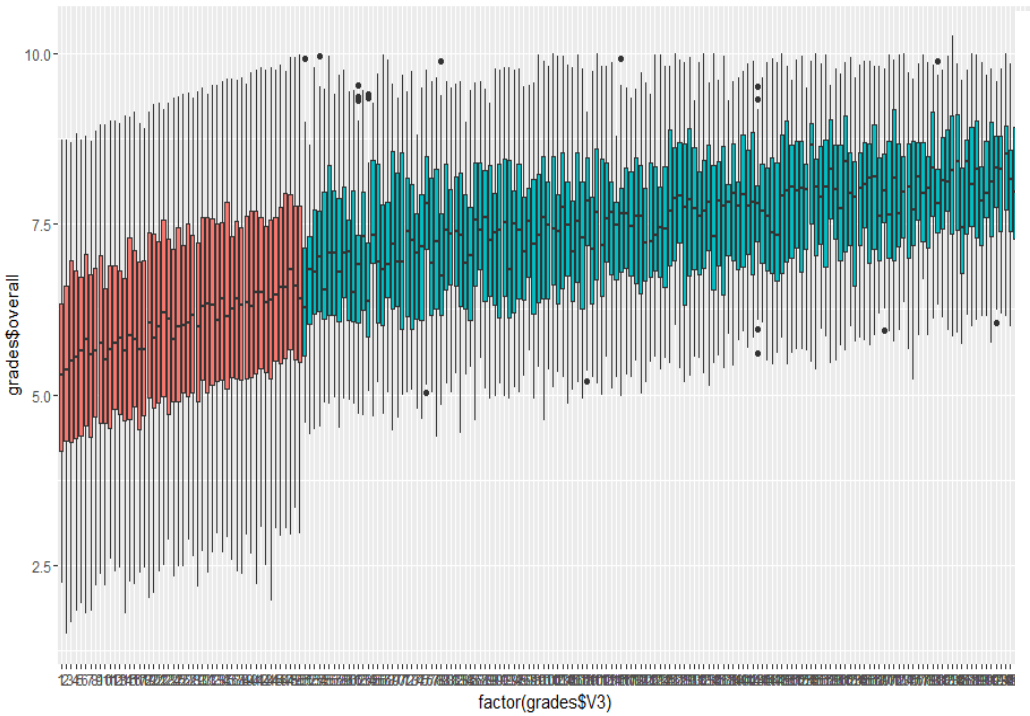
Results from R (1 sample)

- n=1240
- mean: 5.79 (!) (population = 6.52)
- s.e. SRS .045624
- s.e. cluster .17397

- D_{eft}= .17397 /.045624 = 3.81

- Why are results so bad?

Problems in 1-stage cluster sampling



In our example:

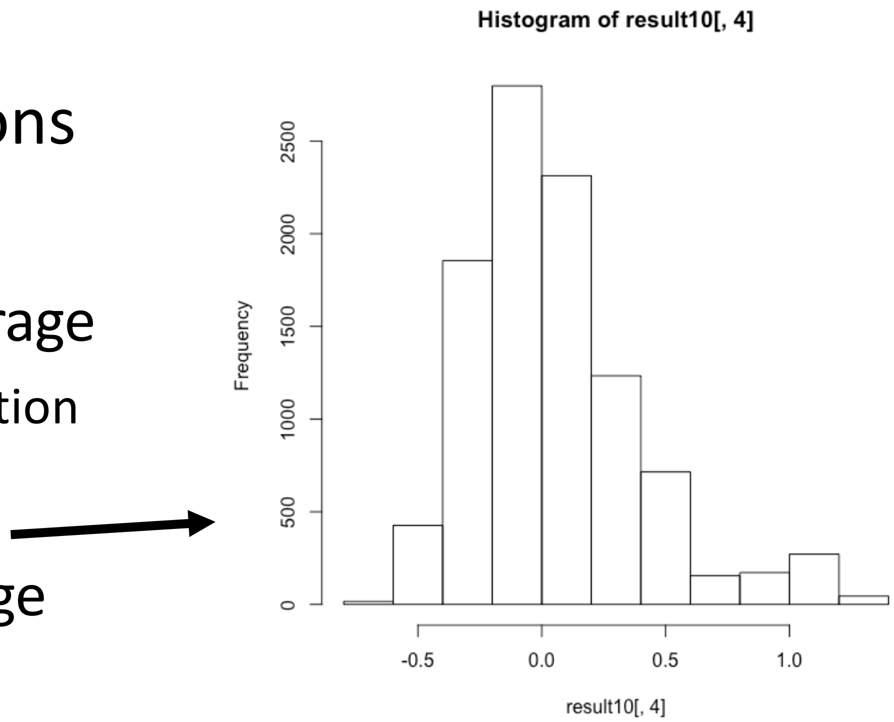
- Bad luck: Clusters selected:
  - 1,3,13,39,56,102,187
- Small no. of clusters sampled
- Large difference between cluster means (large ICC)
  - ICC population:
  - For Ba/MA: .18
  - For Programme: .21

In general:

- hard to control sample size
- Design effect could be large if variance differs across clusters

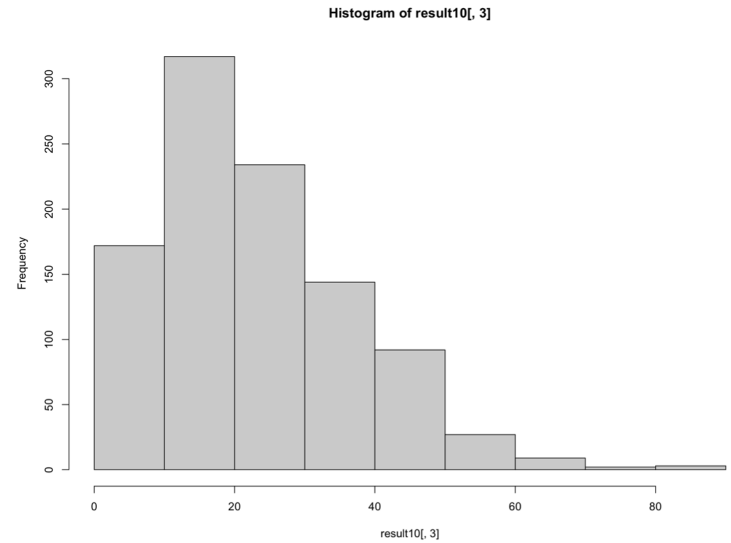
Were we unlucky? a simulation

- 10000 replications
- 10 clusters
  - n=1000 on average
    - but large variation
- Bias
  - ,0005 on average
  - Long tail
    - when we sample only MA or BA



a simulation for 1-stage cluster (2)

- 10000 replications
- 10 clusters
  - n=1000 on average
    - but large variation
- Design effect (D_{eff}):
  - 23 (!) on average
  - We would need sample of 23.000!



Better: A 2-stage cluster sample

- Double the clusters, sample half
  - 1. Sample 20 clusters (instead of 10) using SRS
  - 2. Sample ½ of i within every cluster > expected sample size= 1000
    - Or do 50 clusters and sample $^1/_5$ of individuals, etc.
- Optimal allocation given ICC and costs
  - But need good estimates for:
    - Costs?
    - population total? , cluster sizes? (~fpc),
    - Often: use of “pseudoclusters”

Two-stage cluster simulation

- 10000 replications
- 20/50 clusters
  - Second stage sample ½ or $^1/_5$
  - n=1000 on average (but less variation)
- Bias
  - 20 clusters: -.0034
  - 50 clusters: -.0322

Two-stage cluster simulation

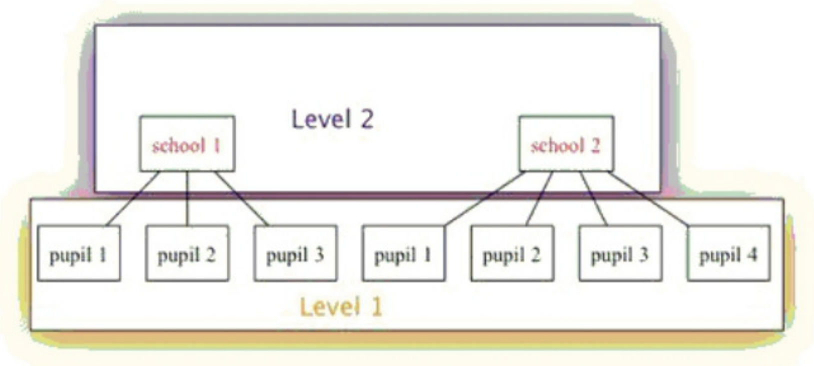
- 10000 replications
- 20/50 clusters
  - Second stage sample ½ or $^1/_5$
  - n=1000 on average (but less variation)
- Design effect (deft)
  - 20 clusters: 16
  - 50 clusters: 6.8

Two-stage cluster estimation

- Uncertainty now at 2 levels
  - Sampling of clusters
  - Sampling of individuals in clusters
- Ok for Means, regression coefficients
  - Imagine we do SRS for selection of clusters and individuals
- Complex for variances
  - Rely on R for variance estimation

Analysis of cluster samples

1. Use the survey package
2. Do multilevel analysis (in semester 2)



Equation for Level 1

$$y_{ij} = \beta_{0j} + \varepsilon_{ij}$$

Equation for Level 2

$$\beta_{0j} = \gamma_{00} + \mu_{0j}$$

3. Economists use Huber-White "robust" standard errors
   1. Pretend your dataset is SRS,
   2. estimate d_{eft} and
   3. multiply your s.e. with deft

Class exercise 2

- Analyse a cluster sample for the “boys” dataset
- See Class exercise document (Blackboard)

- If unfinished, finish at home before practical next week

Next week:

- Finish: Class exercises – different sampling designs
  - discussion about problems in class next week.
- Take Home Exercise:
  - Obtain the data: you often need to register at the data archive
- Next week: practical (in class time)
  - Mix SRS, stratified and cluster sampling
    - Design weights
    - Hurvitz-Thompson estimator
    - Multistage sampling
    - …